

Cosmic age, Statefinder and Om diagnostics in the decaying vacuum cosmology

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Abstract

As an extension of Λ CDM, the decaying vacuum model (DV) describes the dark energy as a varying vacuum whose energy density decays linearly with the Hubble parameter in the late-times, $\rho_\Lambda(t) \propto H(t)$, and produces the matter component. We examine the high- z cosmic age problem in the DV model, and compare it with Λ CDM and the Yang-Mills condensate (YMC) dark energy model. Without employing a dynamical scalar field for dark energy, these three models share a similar behavior of late-time evolution. It is found that the DV model, like YMC, can accommodate the high- z quasar APM 08279+5255, thus greatly alleviates the high- z cosmic age problem. We also calculate the Statefinder (r, s) and the Om diagnostics in the model. It is found that the evolutionary trajectories of $r(z)$ and $s(z)$ in the DV model are similar to those in the kinessence model, but are distinguished from those in Λ CDM and YMC. The $Om(z)$ in DV has a negative slope and its height depends on the matter fraction, while YMC has a rather flat $Om(z)$, whose magnitude depends sensitively on the coupling.

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1 Introduction

The observations of the luminosity-redshift relation $d_L(z)$ of type Ia supernovae have indicated that the expansion of the universe is accelerating [1]. Within the framework of general relativity, this would imply that there exists a cosmic dark energy (DE) as a major component in the Universe that drives the acceleration. Observations of cosmic microwave background (CMB) [2, 3, 4], and large scale structure [5], especially baryon acoustic oscillations [6], weak gravitational lensing [7], and X-ray clusters [8], have also shown that the universe is spatially flat with $\sim 75\%$ of DE and $\sim 25\%$ of matter. Besides the cosmological constant model (Λ CDM) [9], a number of models of DE have been proposed as the potential source for the acceleration. Among them there are various scalar models [10], vector models, such as the Yang-Mills condensate (YMC) [11, 12], and modified gravity [13], and so on.

As an extension to Λ CDM, another model proposed recently is the decaying vacuum (DV) dark energy model without referring to a specific dynamic field [14, 15]. In this model, the DE described by the varying vacuum is decaying and producing the matter as the universe expands, and the equation of state (EOS) of the vacuum is a constant value $w_\Lambda = p_\Lambda(t)/\rho_\Lambda(t) = -1$, the same as that in Λ CDM. One interesting feature of the DV model is that its late-time dynamics is similar to Λ CDM, and to the YMC dark energy models [11, 12]. The DV model has been tested by observational data of supernova Ia, yielding a constraint on the present matter density contrast and the Hubble parameter: $0.27 \leq \Omega_{m0} \leq 0.37$ and $0.68 \leq h \leq 0.72$ (at 2σ) [16]. A joint statistical analysis of SN Ia, the baryonic acoustic oscillation, and the cosmic microwave background anisotropies (CMB), gives the constraint $\Omega_{m0} \leq 0.36 \pm 0.01$ and $h = 0.69 \pm 0.01$ [17]. The similar joint statistical analysis has also been made, with a much larger sample of SN Ia [18], yielding $\Omega_{m0} = 0.289^{+0.013}_{-0.014}$ and $h = 0.634 \pm 0.004$ for the YMC model of 3-loop corrections, and $\Omega_{m0} = 0.283$ and $h = 0.638$ for Λ CDM. The χ^2 statistics has shown that YMC performs slightly better than Λ CDM [19]. In this paper, we will confront the DV model with the high- z cosmic age problem, and examine it by the Statefinder (r, s) and the Om diagnostics, aiming at differentiating it from Λ CDM, YMC, and several other DE models.

The so-called “high- z cosmic age problem” is related to the quasar APM 08279+5255, whose age $t_{qua} = (2.0 - 3.0)$ Gyr at the redshift $z = 3.91$, as evaluated by its chemical evolution [20, 21]. The problem has been examined in several DE models [21, 22, 23]. As is known, the Λ CDM model has difficulty in accommodating the high- z quasar. Ref.[22] has shown that the introduction

of a coupling between DE and matter can generally increase the high- z cosmic age. As an example, it has been found that the YMC dark energy coupled with the matter greatly alleviates the problem. Since the DV scenario is an interacting DE model, it is also expected to have a longer high- z cosmic age.

To distinguish the DV model from other DE models, the Statefinder diagnosis (r, s) can be used [24], which has been applied to several other DE models [25, 26, 27]. As a complementary to (r, s) , a new diagnostic called Om has been recently proposed [28], which helps to distinguish the present matter density contrast Ω_{m0} in different models more effectively. We will explore both (r, s) and Om diagnostics for the DV model, and compare it with Λ CDM and YMC. In the following, we use the unit $8\pi G = c = 1$.

2 The decaying vacuum model

In a spatially flat ($\Omega_{\Lambda 0} + \Omega_{m0} = 1$) Robertson-Walker (RW) space-time, the Friedmann equation is

$$\rho_T = 3H^2, \quad (1)$$

with $H = \dot{a}/a$ being the Hubble rate of expansion, where an overdot means taking derivative with respect to the cosmic time t . The energy conservation equation is given by

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \quad (2)$$

where ρ_T and p_T are the total energy and pressure, respectively. For the present epoch, the universe consists of the matter (baryons and dark matter) and the DE, and the radiation can be ignored. So $\rho_T = \rho_m + \rho_\Lambda$ and $p_T = p_\Lambda$. Moreover, in the DV model, $\rho_\Lambda = \Lambda(t)$ and $p_\Lambda = -\Lambda(t)$. Thus Eq. (2) can be written as

$$\dot{\rho}_m + 3H\rho_m = -\dot{\Lambda}, \quad (3)$$

showing that the decaying vacuum density $\Lambda(t)$ plays the role of a source of matter production. If Λ is a constant, Eq.(3) reduces to the continuity equation for matter in Λ CDM model. To proceed, one takes the so-called late-time ansatz [14, 15]

$$\Lambda(t) = \sigma H(t) \quad (4)$$

where σ is a constant. Then one gets the following solutions [14]

$$a(t) = C(e^{\sigma t/2} - 1)^{2/3} \quad (5)$$

$$\rho_m(t) = \frac{\sigma^2 C^3}{3a^3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}, \quad (6)$$

$$\Lambda(t) = \frac{\sigma^2}{3} + \frac{\sigma^2 C^{3/2}}{3a^{3/2}}, \quad (7)$$

with C being an integration constant. If the normalization of the present scale factor $a(t_0) = 1$ is taken, one can check from Eqs.(6) and (7) that

$$C = \left(\frac{\Omega_{m0}}{1 - \Omega_{m0}} \right)^{2/3}, \quad (8)$$

and $\sigma = 3\Omega_\Lambda H_0$, where $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the present Hubble constant and h is the Hubble parameter. The first terms in Eqs. (6) and (7) give the usual scaling of the matter and the vacuum, respectively, while the second terms describe the matter production caused by the decaying vacuum. It is these interacting terms between the vacuum and the matter [22] that will be likely to increase the high- z cosmic age in the DV model. Eqs.(5), (6) and (7) give a smooth transition between the matter with $a \ll 1$ and vacuum epoch with $a \gg 1$. In the following, we will examine the high- z cosmic age problem and the Statefinder and Om diagnostics in the DV model.

3 High- z cosmic age problem

As is known, introducing the cosmological constant Λ can increase the predicted cosmic age. Similarly, the presence of $\Lambda(t)$ in the DV model also increases the present cosmic age with $t_0 \sim 15 \text{ Gyr}$ [17], in concordance with the current estimated one $t_0 \sim 14 \text{ Gyr}$ from a combination of data of Wilkinson Microwave Anisotropy Probe (WMAP) and Sloan Digital Sky Survey (SDSS) [29]. However, the high- z cosmic age problem is more difficult to solve. For the quasar APM 08279+5255, the estimated age $t_{qua} = (2.0 - 3.0) \text{ Gyr}$ at $z = 3.91$ has been based upon the observed high Fe/O abundance ratio ~ 3 from the X-ray data [20] and upon the chemodynamical modeling for the evolution [30]. For instance, in a detailed investigation, Ref.[21] gives an age $t_{qua} = 2.1 \text{ Gyr}$ at $z = 3.91$. For a cosmological model to solve the high- z problem, its predicted cosmic age at $z = 3.91$ should be older than the lower limit of the observed age of 2.0 Gyr.

Given a cosmological model, the cosmic age $t(z)$ as a function of redshift z is written as

$$t(z) = \int_z^\infty \frac{d\tilde{z}}{(1 + \tilde{z})H(\tilde{z})}, \quad (9)$$

where $H(z)$ is the Hubble parameter in terms of redshift z . For the DV model, using Eqs. (1) and (5)-(7), one obtains

$$H(z) = H_0[1 - \Omega_{m0} + \Omega_{m0}(1 + z)^{3/2}]. \quad (10)$$

For comparison, we also list

$$H(z) = H_0[1 - \Omega_{m0} + \Omega_{m0}(1 + z)^3]^{1/2} \quad (11)$$

for the Λ CDM model, and

$$H(z) = H_0[(1 - \Omega_{m0})\frac{\rho_y(z)}{\rho_{y0}} + \Omega_{m0}\frac{\rho_m(z)}{\rho_{m0}}]^{1/2} \quad (12)$$

for the YMC DE model [11, 12, 19, 22]. In Eq.(12), $\rho_y(z)$ and $\rho_m(z)$ are the energy density of the YMC and of the matter, respectively, which depend implicitly on the coupling between the YMC and the matter. Here we will consider the case of 2-loop YMC decaying into the matter at a constant decay rate $\Gamma \sim H_0$ [12].

Substituting Eqs.(10), (11), and (12), respectively, into Eq.(9), yields the cosmic age $t(z)$ predicted by the DV, the Λ CDM, and the YMC model. Since $H(z)$ depends explicitly the parameters h and Ω_{m0} , so does the cosmic age $t(z)$ given by Eq.(9). With other parameters fixed, a larger h tends to yield a smaller $t(z)$, and so does a larger Ω_{m0} . Currently, the value of h is still in debate even after a number of experiments and observations of various kinds in the past.

For instance, the final result from the Hubble Space Telescope (HST) Key Project [31] is $h = 0.72 \pm 0.08$, and Riess et al. recently give $h = 0.742 \pm 0.036$ [32]. Whereas Sandage et al. [33] advocate the global value $h = 0.623 \pm 0.063$ from HST using SN Ia, calibrated with Cepheid variables in nearby galaxies. From the observational data on CMB anisotropies, WMAP3 [3] and WMAP5 [4] give $h = 0.732 \pm 0.031$ and $h = 0.701 \pm 0.013$, respectively. On the other hand, the value of Ω_{m0} is less uncertain, and has been determined by WMAP3 [3], WMAP5 [4], and SDSS [29], to be 0.24, 0.25, and 0.24, respectively.

It is convenient to use a dimensionless cosmic age $T(z) \equiv H_0 t(z)$, which depends on Ω_{m0} only. For three DE models with $\Omega_{m0} = 0.27$: DV, Λ CDM, and 2-loop YMC [12], Fig.1 shows the respective $T(z)$. Besides, $T(z)$ of DV model with $\Omega_{m0} = 0.32$ is also plotted. The dimensionless age $T_{qua} \equiv H_0 t_{qua}$ of the quasar APM 08279+5255 depends on the parameter h . With $t_{qua} = (2.0 - 3.0)$ Gyr, T_{qua} has a range of (0.147, 0.221) for $h = 0.72$ and of (0.127, 0.190) for $h = 0.62$ at $z \sim 3.91$. These are represented by the vertical line segments in Fig. 1.

The Universe can not be younger than its constituents at any redshift. Specifically, in regard to the quasar APM 08279+5255, the cosmic age at $z = 3.91$ predicted in a cosmological model must be greater than the quasar age,

$$t(3.91) \geq t_{qua}, \quad i.e., \quad T(3.91) \geq T_{qua}. \quad (13)$$

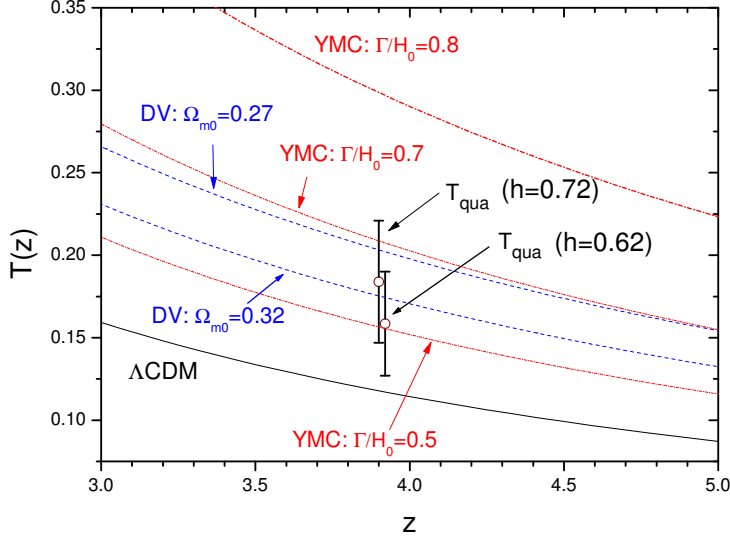


Figure 1: The dimensionless cosmic age $T(z) \equiv H_0 t(z)$ are plotted for three models. For the APM 08279+5255 with age $t_{qua} = (2.0 - 3.0)$ Gyr, the two vertical line segments at $z \sim 3.91$ indicate $T_{qua} = (0.147, 0.221)$ for $h = 0.72$, $T_{qua} = (0.127, 0.190)$ for $h = 0.62$. Here $\Omega_{m0} = 0.27$ is taken for Λ CDM and YMC.

Fig.1 shows that, for either $h = 0.72$ or 0.62 , the cosmic age $t(z)$ predicted by DV lies safely above the lower limit $t_{qua} = 2\text{Gyr}$ at $z = 3.91$. For $h = 0.62$ and $\Omega_{m0} = 0.27$, $t(z)$ is even higher than the upper limit $t_{qua} = 3\text{Gyr}$ at $z = 3.91$. Thus we can say that the DV model greatly alleviates the high- z cosmic age problem. Fig.1 also shows that a low Ω_{m0} yields a higher $t(z)$. The reason that DV has a higher cosmic age is that the model intrinsically has an interaction between the DE and the matter. These interaction terms are fully determined by H_0 and Ω_{m0} , and are not adjustable. On the other hand, for YMC the interaction is realized by the decay rate Γ as a model parameter, and a higher rate Γ will yield a longer comic age. For instance, the YMC with $\Gamma = 0.7H_0$ performs a bit better than the DV, and has an age $T(z)$ near the upper limit T_{qua} even for $h = 0.72$. As shown in Fig.1, the Λ CDM cannot accommodate the quasar even for $h = 0.62$.

4 Statefinder and Om diagnostics in the DV model

The Statefinder diagnostic pair $\{r, s\}$ are defined as [24]

$$r \equiv \frac{\ddot{a}}{aH^3}, \quad s \equiv \frac{r - 1}{3(q - 1/2)}, \quad (14)$$

where $q = \frac{\ddot{a}}{aH^2}$ is the deceleration parameter. These parameters can be also expressed in terms of ρ_T and p_T as follows

$$q = \frac{1}{2}\left(1 + \frac{3p_T}{\rho_T}\right), \quad r = 1 + \frac{9(\rho_T + p_T)\dot{p}_T}{2\rho_T\dot{\rho}_T}, \quad s = \frac{\rho_T + p_T}{p_T} \frac{\dot{p}_T}{\dot{\rho}_T}. \quad (15)$$

For the DV model, one has

$$q = \frac{3}{2}\Omega_m - 1, \quad (16)$$

$$r = 1 - \frac{9}{4}\Omega_m(1 - \Omega_m), \quad (17)$$

$$s = \frac{1}{2}\Omega_m, \quad (18)$$

where $\Omega_m = \rho_m(t)/\rho_T(t)$ is the time dependent matter fraction. By Eqs. (5)–(7) it can be written as

$$\Omega_m = e^{-\sigma t/2} = \frac{\Omega_{m0}(1+z)^{3/2}}{[1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}]}. \quad (19)$$

Substituting it into Eqs. (16)–(18) yields

$$q = \frac{3\Omega_{m0}(1+z)^{3/2}}{2[1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}]} - 1, \quad (20)$$

$$r = 1 - \frac{9\Omega_{m0}(1 - \Omega_{m0})(1+z)^{3/2}}{4[1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}]^2}, \quad (21)$$

$$s = \frac{\Omega_{m0}(1+z)^{3/2}}{2[1 - \Omega_{m0} + \Omega_{m0}(1+z)^{3/2}]}, \quad (22)$$

which depend on the parameter Ω_{m0} . We plot the parameters $r(z)$ and $s(z)$ in Fig.2 for the DV model with $\Omega_{m0} = 0.27$ and with $\Omega_{m0} = 0.32$, respectively. For comparison, $r(z)$ and $s(z)$ in Λ CDM, YMC, quiescence, kinessence models [24] are also plotted there. The left panel of Fig.2 shows that $r(z)$ in the DV model has a minimum at $z \sim 1$, a feature drastically distinguished from other models. Since this feature appears around $z \sim 1$, where observational data are easier to obtain, direct confrontations of the DV model with observations is feasible. Moreover, the right panel of Fig.2 shows that the behavior of $s(z)$ in the DV model is similar to that in the kinessence model, different from other models. In particular, it should be pointed out that although DV and Λ CDM have the same EOS $w = -1$, their Statefinder pair (r, s) are effectively distinguished. On the other hand, amongst the various models plotted, YMC has a Statefinder $r(z), s(z)$ that is most close to that of Λ CDM with $(r, s) = (1, 0)$.

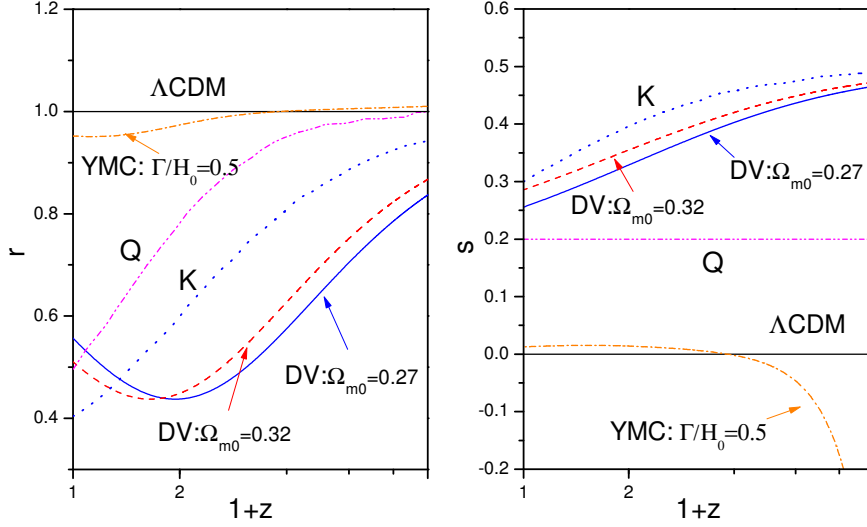


Figure 2: $r(z)$ and $s(z)$ in the DV model for two cases with $\Omega_{m0} = 0.27$ and 0.32 . For comparison, the results are also plotted for other DE models including the Λ CDM, the coupling YMC with a decay rate $\Gamma = 0.5H_0$, quiescence (Q), and kinessence (K) models [24].

The trajectories of (r, s) and of (r, q) can also help to differentiate various DE models. The results of DV, YMC, Λ CDM are plotted in Fig.3. These two trajectories distinguish the DV from Λ CDM and YMC very effectively. The shape of the trajectories of (r, s) of DV are quite similar to those of kinessence model given by Fig.1 in Ref.[24], which, nevertheless, has a time dependent EOS $w \neq \text{constant}$. Also notice that the shape of the curve $s - r$ is quite similar to that of the curve $q - r$. This is easy to understand since q and s are linearly related by $q = 3s - 1$, according to Eqs. (20) and (22). From Fig. 2 and Fig. 3 we notice that the Statefinder (r, s) for DV is not very sensitive to the matter fraction Ω_{m0} . For instance, for $\Omega_{m0} = 0.27$ and $\Omega_{m0} = 0.32$, the left panel in Fig. 3 shows almost overlapping trajectories (r, s) , and the right panel shows almost overlapping trajectories (r, q) . This kind of degeneracy can be broken to certain extent by the Om diagnostic as follows.

Now we turn to the Om diagnostic defined as [28]

$$Om(x) = \frac{h^2(x) - 1}{x^3 - 1}, \quad (23)$$

where $x \equiv (1 + z)$ and $h(x) \equiv H(x)/H_0$. Thus Om involves only the first derivative of the scale factor through the Hubble parameter and is easier to reconstruct from observational data. For Λ CDM with Eq.(11), it is simply

$$Om(x) = \Omega_{m0}, \quad (24)$$

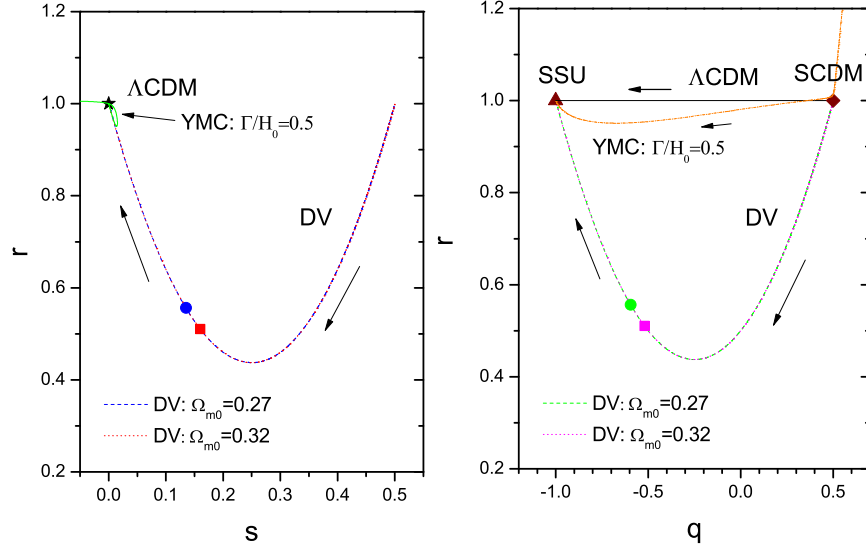


Figure 3: The trajectories of (r, s) and of (r, q) in the DV model for two cases: $\Omega_{m0} = 0.27$ and $\Omega_{m0} = 0.32$. The filled circles and squares stand for the cases of $\Omega_{m0} = 0.27$ and $\Omega_{m0} = 0.32$, respectively. The star in the $r - s$ plane means the fixed point of Λ CDM. The diamond and the triangle in the $r - q$ plane denote the fixed points of the standard cold dark matter (SCDM) model and of the steady state Universe (SSU) [27], respectively. The arrows along the curves are the evolution direction.

independent of redshift. For DV with Eq. (10), one has

$$Om(x) = \frac{(1 - \Omega_{m0} + \Omega_{m0}x^{3/2})^2 - 1}{x^3 - 1}. \quad (25)$$

For YMC, one has

$$Om(x) = \frac{[(1 - \Omega_{m0})\frac{\rho_y(x)}{\rho_{y0}} + \Omega_{m0}\frac{\rho_m(x)}{\rho_{m0}}]^2 - 1}{x^3 - 1}. \quad (26)$$

Fig. 4 shows $Om(z)$ for $z \leq 2.5$ in DV and YMC, both being distinguished from Λ CDM. $Om(z)$ in DV has a negative slope, similar to that quintessence models. Moreover, the Om diagnostic distinguishes effectively the two cases $\Omega_{m0} = 0.27$ and $\Omega_{m0} = 0.32$ of the DV model. This is an advantage of Om over the Statefinder diagnostic. Besides, although $Om(z)$ are quite flat in both YMC and Λ CDM, $Om(z)$ in YMC lies far below that of Λ CDM. So they are also differentiated more easily by the Om diagnostic than by the Statefinder diagnostic [26, 19]. Furthermore, for a fixed Ω_m , the $Om(z)$ in the DV model approaches that in the Λ CDM model as $z \rightarrow 0$, but the flat $Om(z)$ in YMC does not have that property.

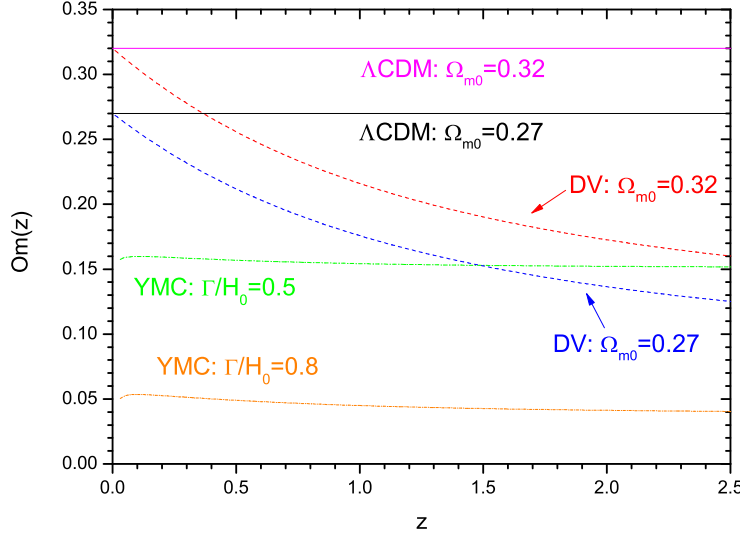


Figure 4: $Om(z)$ in DV has a negative slope and its height depends sensitively on Ω_{m0} . $Om(z)$ in YMC is rather flat and its height depends on the coupling Γ .

5 Summary

We have demonstrated that, the DV dark energy model with a low value of Ω_{m0} can marginally accommodate the high- z quasar APM 08279+5255, if the Hubble parameter $h = 0.72$ or smaller. This is due to the fact that the decaying $\Lambda(t)$ intrinsically induces an interaction with the matter component, thus increasing the cosmic age. To solve the age problem completely, a lower value $\Omega_{m0} \leq 0.27$ would be required. But observational data of SN Ia has already put a constraint $0.27 \leq \Omega_{m0} \leq 0.36$ on the DV, so one can only say that the DV alleviates the high- z cosmic age problem. In comparison, the interacting YMC model can solve the problem by increasing the decay rate Γ .

To examine DV further, we have also calculated the Statefinder diagnostic (r, s) in the model, and have made comparison with Λ CDM, YMC, quiescence, and kinessence. It has been found that (r, s) effectively distinguishes DV from Λ CDM, even though the two models have the same EOS $w = -1$. Besides, (r, s) in DV is also different from those in YMC, quiescence, and kinessence. The trajectories of the pair (r, s) and (r, q) have a similar shape in DV because of a linear relation between s and q .

While the Statefinder (r, s) for DV is not very sensitive to Ω_{m0} , the Om diagnostic has been employed to break this degeneracy. It has been shown that the $Om(z)$ in DV has a negative slope and its height depends on the matter fraction, while YMC has a rather flat $Om(z)$, which depends sensitively on

the magnitude of the coupling. Therefore, the Om diagnostic will be quite effective in distinguishing Λ CDM, DV, and YMC models from observational data of relatively low redshifts.

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